Problem Set 3A: Assignment 3

Instructor: El Mehdi Ainasse
MAT 342 – Applied Complex Analysis
Summer Session II 2019

DUE: August 1st, 2019 – AT THE BEGINNING OF CLASS.

Exercise 0. Review everything you've studied this week before proceeding!

Exercise 1. Let $f(z) = \dfrac{e^{z^2}}{(2-z)^2}$.

1. What is the value of the integral of f over any simple closed curved inside the unit semi-circle?
2. What is the value of the integral of f over the circle of radius 2 centered at the origin?

Exercise 2. Suppose that f is holomorphic on the disk $\{|z| < 2\}$ and satisfies $\left|f(z)\right| < \sqrt{2}$. What estimate can be made about $\left|f'(0)\right|$?

Exercise 3. Evaluate the integral

$$\int_\gamma \left(\left(\mathrm{Re}(z)\right)^2 + i\left(\mathrm{Im}(z)\right)^2\right) dz,$$

where γ is the line joining 1 to i.

Exercise 4. Let f be an entire function, and write $f(x+iy) = u(x,y) + iv(x,y)$. Suppose that $u(x,y) + v(x,y) \geq 1$. Let $\varphi(z) = e^{-(1-i)f(z)}$.

1. Is the function φ entire? Is it bounded?
2. What can you say about f based on your answer to (a)?

Exercise 5. Prove that $g(z) = \displaystyle\sum_{n=1}^{\infty} \frac{1}{z^n}$ defines a holomorphic function for $|z| > 1$.