Problem Set 2B: Practice Problem Set 2

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MAT 342 – Applied Complex Analysis
Summer Session II 2019

NEVER DUE. Do the exercises for your own benefit. Practice makes perfect. On this note, keep in mind that the assignments are mostly for grading purposes and are thus not enough practice. If you have any questions, let me know.

Exercise 0. Review everything you've studied this week before proceeding!

Exercise 1. Suppose that f is analytic for $|z| < 2$ and let α be a complex constant. Show that:

$$\int_{\{|z|=1\}} \left(\mathrm{Re}(z) + \alpha\right) \frac{f(z)}{z} dz = \int_{\{|z|=1\}} \frac{1}{2} \left(\frac{1}{z^2} + \frac{2\alpha}{z}\right) f(z) dz.$$

Exercise 2. Suppose f is a continuous function on the unit circle which is bounded by some positive real M, i.e. $\left|f(e^{i\theta})\right| \leq M$. Suppose also that

$$\left|\int_{\{|z|=1\}} f(z) dz\right| = 2\pi M.$$

Show that $f(z) = c\bar{z}$ for some constant $c \in \mathbb{C}$ such that $|c| = M$.

Exercise 3. Let $n, m \geq 1$ and $R > 1$. Show that:

$$\left|\int_{\{|z|=R\}} \frac{z^n}{z^m - 1} dz\right| \leq \frac{2\pi R^{n+1}}{R^m - 1}.$$

What can you say about the value of the integral as R gets very large (i.e. tends to ∞)?

Exercise 4. Let $f(z) = az^2 + b|z|^2 + c\bar{z}^2$ where $a, b, c \in \mathbb{C}$ are fixed. Using the limit-definition of holomorphicity, find the values of a, b and c for which f is holomorphic.

Exercise 5. Let p be the polynomial defined by $p(z) = a_n z^n + a_{n-1} z^{n-1} + \cdots + a_1 z + a_0$. Show that for any radius $R > 0$ and $z_0 \in \mathbb{C}$:

$$p(z_0) = \frac{1}{2\pi i} \int_{\{|z - z_0| = R\}} \frac{p(z)}{z - z_0} dz.$$