

## Problem Set 2A: Assignment 2

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MAT 342 – Applied Complex Analysis  
Summer Session II 2019

**DUE: July 25th, 2019 – AT THE BEGINNING OF CLASS.**

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**Exercise 0.** Review everything you've studied this week before proceeding!

**Exercise 1.** Compute the complex log of each of the numbers  $2, i, 1 + i$  and  $\frac{1 + i\sqrt{3}}{2}$ .

**Exercise 2.** 1. Using the geometric series, show that for  $z \neq 1$ :

$$1 + 2z + 3z^2 + \cdots + nz^{n-1} = \frac{1 - z^n}{(1 - z)^2} - \frac{nz^n}{1 - z}.$$

2. Show that  $e^{\bar{z}} = \overline{e^z}$ .

**Exercise 3.** Let  $f$  be holomorphic on  $A$  and let  $A^* = \{z \in \mathbb{C} : \bar{z} \in A\}$  be the set of points  $\mathbb{C}$  whose conjugates are in  $A$ . Suppose  $f$  is holomorphic on  $A$ .

1. Show that the function  $g$  defined by  $g(z) = \overline{f(\bar{z})}$  is holomorphic on  $A^*$ .

2. Show that  $g'(z) = \overline{f'(\bar{z})}$ .

**Exercise 4.** Compute the length of the contour defined by  $\gamma(t) = e^t (\cos(t) + i \sin(t))$  for  $-\pi \leq t \leq \pi$ .

**Exercise 5.** Compute the integral

$$\int_{\{|z|=1\}} \left( \frac{z \sin(z)}{z + 2} + \bar{z} \right) dz.$$