Problem 1.

Suppose that A and B are non-empty finite sets of real numbers such that $A \subseteq B$.
Prove that:

$$\min(B) \leq \min(A) \leq \max(A) \leq \max(B)$$

Answer:

(You may use the back of this page if necessary.)

Problem 2.

(a) Let $f : X \to \mathbb{N}_n$ be an injection. Prove then that X is finite and $|X| \leq n$.

(b) Given non-empty finite sets X and Y with $|X| < |Y|$, prove that there does not exist a surjection $X \to Y$.
You can assume the following fact:
If $f : \mathbb{N}_n \to X$ is a surjection, then X is finite and $|X| \leq n$.

Answer:

(You may use the back of this page if necessary.)

Problem 3. Prove that there does not exist a rational number whose square is 10.

Answer:
(You may use the back of this page if necessary.)

Problem 4.

Use the Euclidean algorithm to find the greatest common divisor of 165 and 252.

Answer:

(You may use the back of this page if necessary.)

Problem 5.

Let n be an integer.

(a) Prove that n^2 is divisible by 5 **if and only if** n is divisible by 5.

(b) Use the result of (a) to prove that there does not exist a rational number number whose square is 5.

Answer:

(You may use the back of this page if necessary.)

Problem 6.

(a) Let a be an integer. Prove that if a is even, then a^2 is even, and if a is odd, then $a^2 = 4p + 1$ for some $p \in \mathbb{Z}$.

(b) Prove that an integer n is the sum of two squares ($n = a^2 + b^2; a, b \in \mathbb{Z}$), then $n = 4q$ or $n = 4q + 1$ or $n = 4q + 2$ for some $q \in \mathbb{Z}$.

Answer:

(You may use the back of this page if necessary.)