Problem 1.

For any propositions P and Q, write the truth tables for each of the propositions '$P \Rightarrow Q$' and '$\neg P \vee Q$'. Are these two propositions ('$P \Rightarrow Q$' and '$\neg P \vee Q$') equivalent?

Answer:

(You may use the back of this page if necessary.)

Problem 2. We define a integer m to be the *smallest* number if it is smaller (lesser) than any other *integer*.

(a) Rewrite the sentence 'm is smaller than any other integer' in mathematical symbols using quantifiers if necessary.

(b) Prove that there exists no such number via a proof by contradiction.

Answer:

(You may use the back of this page if necessary.)

Problem 3. Prove by induction that $\prod_{i=2}^{n}\left(1-\frac{1}{i^2}\right)=\frac{n+1}{2n}$ for all $n \geq 2$.

Answer:

(You may use the back of this page if necessary.)

Problem 4. Define the function $f : \mathbb{N}^2 \longrightarrow \mathbb{N}^2$ by $f(n, m) = (n + m, n - m)$.

(a) Prove that f is injective.

(b) Is f bijective? If so, prove it. If not, explain why and provide a counter-example.

Answer:

(You may use the back of this page if necessary.)

Problem 5.

Let A, B and C. Prove that:

$$A \times (B \cup C) = (A \times B) \cup (A \times C)$$

Answer:

(You may use the back of this page if necessary.)

Problem 6.

Define the characteristic function of a set A as being the function such that:
$\chi_A(x) = 0$ if $x \notin A$ and $\chi_A(x) = 1$ if $x \in A$.

(a) Prove that $\chi_A(x) = 1 - \chi_{A^c}(x)$ for any set A. A^c denotes the complement of the set A.

(b) Prove that $\chi_{A \cap B}(x) = \chi_A(x)\chi_B(x)$ for any sets A and B.

(c) Prove that $\chi_{A \cup B}(x) = \chi_A(x) + \chi_B(x) - \chi_A(x)\chi_B(x)$ for any sets A and B.

Answer:

(You may use the back of this page if necessary.)