Midterm I Review Sheet

Instructor: El Mehdi Ainasse
MAT 200 - Language, Proof and Logic
Summer Session I 2017

General remark: As you practice problems, you should focus on Problems I (p. 53) and Problems II (p. 115).

Part I – Chapters 1 – 5:
I think that the following notes by Ben Chow do a great job at summarizing efficiently the contents of these chapters:
`http://www.math.ucsd.edu/~benchow/Week1notes.pdf`
`http://www.math.ucsd.edu/~benchow/Week2notes.pdf`

Remarks: In practice, you would only need strong induction if the induction statement involves more than just n. For example: "For any integer odd n, both $n-1$ and $n+1$ are divisible by 2." This statement depends not only n but also on $n-1$ and $n+1$. Note also that even when applying (weak) induction, it is sometimes necessary to prove more than one base case. (We've talked about this in class.)

Specific pages to target for review:
Chapter 1: all of it. (Just skim through it.)
Chapter 2: p. 11 (Table 2.1.1), pp. 14-15 (***Reading implications***).
Chapter 3: pp. 22-23, 25-28 (*Proof by cases* onwards).
Chapter 4: pp. 32-37.
Chapter 5: all of it. (Also nicely summarized in the second link above.)

Part II – Chapters 6 – 9:

Specific pages to target for review:
Chapter 6: p. 62 (learn the symbols for all the sets mentioned and other notations like $\in$ and $\notin$), pp. 63-67, pp. 70-72.
Chapter 7: pp. 75-83 (and here focus on how to translate a symbolic mathematical statement to a statement in words and vice versa), pp. 83-86 (Cartesian products).
Chapter 8: pp. 89-96 (just before sequences), pp. 97-99 (**8.4** onwards; to help you have a better grasp of how domains and codomains work for functions).
Chapter 9: pp. 101-107 (you can ignore the part about inverses of bijective functions).

Remarks: It is NEVER okay to prove things about sets using Venn diagrams. These usually assume *something* about the sets (maybe that they're non-empty, or maybe that their intersection isn't, etc.) and can therefore not cover all cases. That's the whole point of having a theory for sets. The diagrams good for intuition only. To prove things, you need to write down an actual *proof*.

To prove that two sets are equal, you can do that directly through equivalences without having to prove a double inclusion. So to show that $A = B$ (at large – so these could be unions of other sets, intersections, cartesian products, etc.), instead of showing that $A \subseteq B$ (by showing that $x \in A \Rightarrow [...] \Rightarrow x \in B$) and that $B \subseteq A$ (by showing that $x \in B \Rightarrow [...] \Rightarrow x \in A$); you can proceed to directly show that $x \in A \Leftrightarrow [...] \Leftrightarrow x \in B$. (Here the "[...]" denote the intermediate steps. This almost always works when you're only using the definitions essentially in your proof and no other properties about sets.

Review the Lecture 4 Notes (posted on Blackboard) for Chapter 9.

In addition to all of this, here's a nice list of distribution laws and whatnot pertaining to both Part I and Part II:

`http://www.cs.um.edu.mt/gordon.pace/Teaching/DiscreteMaths/Laws.pdf`

In these notes, the '≡' sign denotes that the given propositions are equivalent – it's used to distinguish the equivalence relation '⇔', which relates two propositions, and the *fact* that two given propositions are logically equivalent (which is commonly checked using truth tables).