Problem Set 4A: Assignment 4 – Solutions

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MAT 123 - Precalculus
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Exercise 0. Review sections 3.5 & 4.1 – 4.4. Re-read everything thoroughly. Done? Good! You may now move to **Exercise 1**.

Exercise 1. About how many years will it take for a sample of cesium-137, which has a half-life of 30 years, to have only 3% as much cesium-137 as the original sample?

Solution: The equation for the amount of the sample is:

$$a(t) = a_0 \cdot (1/2)^{t/30} = 2^{-t/30}$$

Here a_0 is the original amount. If the current amount is $a(t) = 3\% a_0 = 0.03a_0$, then:

$$0.03a_0 = a_0 \cdot 2^{-t/30}$$

$$0.03 = 2^{-t/30}$$

$$\log_2(0.03) = \log_2(2^{-t/30}) = -t/30$$

$$t = -30\log_2(0.03) \approx 151.7668$$

Exercise 2. Find the coordinates of two points, one on the horizontal axis and one on the vertical axis, such that the distance between these two points equals 15.

Solution: The point on the horizontal axis will have a 0 y-coordinate and so its coordinates are the form $(x_1, 0)$ for some number x. The point on the vertical axis will have a 0 x-coordinate and so its coordinates are of the form $(0, y_2)$.

The distance between the points is then $\sqrt{x_1^2 + y_2^2} = 15$, which means that $x_1^2 + y_2^2 = 225$.

And then a multitude of choices for x_1 and y_2 would work. The easiest choice would be to pick $x_1 = 0$ and $y_2 = 0$ in which case we have $y_2^2 = 225$ or $x_1^2 = 225$ and then $x_1 = 0$ and $y_2 = \pm 15$ or $x_1 = \pm 15$ and $y_2 = 0$.
However, as mentioned, there are many *many* possible answers.

Exercise 3. Sketch the trapezoid whose vertices are (2,1), (6,1), (8,4) and (1,4). Also, find its area.

I'll leave the sketch to you. The small base is $b_1 = 4$, the big base is $b_2 = 7$ and the height is $h = 3$. So the area is $\frac{1}{2}(b_1 + b_2)h = \frac{1}{2}(4+7)\cdot 3 = \frac{11}{6}$

Exercise 4. Let $f(x) = 4 + \ln(x-2)$. Find the domain of f and find a formula for $f^{-1}(x)$.

Solution:

$$f(x) = y$$
$$4 + \ln(x-2) = y$$
$$\ln(x-2) = y - 4$$
$$x - 2 = e^{y-4}$$
$$x = 2 + e^{y-4}$$

And so: $f^{-1}(x) = 2 + e^{x-4}$.

Exercise 5. Find the number c such that the area of $1/x$ on the interval from 5 to c is 4.

Solution: The area under the graph of $1/x$ from 5 to c is $\ln\left(\frac{c}{5}\right)$. We then have that:

$$\ln\left(\frac{c}{5}\right) = 4$$
$$\frac{c}{5} = e^4$$
$$c = 5e^4$$