Problem Set 3A: Assignment 3 – Solutions

Instructor: El Mehdi Ainasse
MAT 123 - Precalculus
Summer Session II 2018

DUE DATE: NEVER.

Exercise 0. Review sections 3.1 – 3.4. Re-read everything thoroughly. Done? Good! You may now move to **Exercise 1**.

Exercise 1. Show that $\sqrt{23 - 8\sqrt{7}} = 4 - \sqrt{7}$.

Solution:

$$(4 - \sqrt{7})^2 = 4^2 - 2 \cdot 4 \cdot \sqrt{7} + \sqrt{7}^2 = 16 - 8\sqrt{7} + 7 = 23 - 8\sqrt{7}$$

And so the result follows.

Exercise 2. Suppose y and b are positive but also that $b \neq 1/2$ and that $b \neq 1$. Show that:

$$\log_{2b} y = \frac{\log_b y}{1 + \log_b 2}$$

Solution:

$$\frac{\log_b y}{1 + \log_b 2} = \frac{\log_b(y)}{\log_b(b) + \log_b(2)} = \frac{\log_b(y)}{\log_b(2b)} = \log_{2b} y$$

The last equality is due to the change of base for log.
(You can check the book. We also talked about it in class.)

Exercise 3. Explain why

$$\log(\sqrt{10x}) = \frac{1 + \log(x)}{2}$$

Solution:

$$\log(\sqrt{10x}) = \frac{1}{2} \log 10x = \frac{1}{2} (\log(10) + \log(x)) = \frac{1 + \log(x)}{2}$$

Exercise 4. Suppose a saving account pays 5% interest per year, compounded four times per year. If the savings account starts with \$600, how many years would it take for the savings account to exceed \$1400?

Solution: As we know, the equation for the amount at a time t is:

$$A(t) = A_0 \cdot \left(1 + \frac{r}{n}\right)^{nt}$$

Here $A_0 = 600$, the initial amount, the time t is in years, and $n = 4$ because the amount is compounded *four* times a year.
So the equation is:

$$A(t) = 600 \cdot \left(1 + \frac{0.05}{4}\right)^{4t}$$

Or, written more simply:

$$A(t) = 600 \cdot (1.0125)^{4t}$$

Let us find the time at which the amount would be 1400:

$$1400 = 600(1.0125)^{4t}$$

$$\frac{1400}{600} = (1.0125)^{4t}$$

$$\frac{7}{3} = (1.0125)^{4t}$$

$$\log_{1.0125}(7/3) = \log_{1.0125}\left((1.0125)^{4t}\right) = 4t$$

$$t = \frac{\log_{1.0125}(7/3)}{4} \approx 17.0517...$$

So the time has to exceed 17 years.

Exercise 5. Solve the following equation:

$$\log_5(x+5) - \log_5(x-1) = 2$$

Solution:

$$\log_5\left(\frac{x+5}{x-1}\right) = 2$$

$$5^{\log_5\left(\frac{x+5}{x-1}\right)} = \frac{x+5}{x-1} = 5^2 = 25$$

$$x + 5 = 25(x-1) = 25x - 25$$

$$-24x = -30$$

$$x = \frac{-30}{-24} = \frac{5}{4}$$