Problem Set 1A: Assignment 1

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MAT 123 - Precalculus
Summer Session II 2018

DUE DATE: July 18th, 2018 – AT THE START OF THE LECTURE.

Exercise 0. Review sections 1.1 – 1.6. Re-read everything thoroughly. Done? Good! You may now move to **Exercise 1**.

Exercise 1. Let $f(x) = \frac{x+2}{x^2+1}$. Find $f(2x^2+3)$. Show your work.

Solution:

$$f(2x^2+3) = \frac{(2x^2+3)+2}{(2x^2+3)^2+1} = \frac{2x^2+5}{(2x)^2+12x^2+9+1} = \frac{2x^2+5}{4x^4+12x^2+10}$$

Exercise 2. Let f be the function whose domain is **the positive numbers** and which has the expression $f(x) = 2x-1$. Let g be the function whose domain is (b, ∞) where b is some real number, and whose expression is $g(x) = \frac{4x^2-1}{2x+1}$. For which number b are the functions f and g equal? Explain your answer.

Solution: $g(x)$ is only defined when $2x+1 \neq 0$, that is when $x \neq -1/2$. Therefore, g can be defined on $(-\infty, -1/2) \cup (-1/2, \infty)$. Since we are told that the domain of g takes the form (b, ∞), it has to fit inside $(-1/2, \infty)$ and so x must be greater (strictly) than $-1/2$. In that case, we may simplify $g(x)$ as follows:

$$g(x) = \frac{4x^2-1}{2x+1} = \frac{(2x-1)(2x+1)}{2x+1} = 2x-1$$

The latter is the same as the expression for $f(x)$. Now f is defined for x in $(0, \infty)$ on which g is also defined since $(0, \infty)$ fits inside $(-1/2, \infty)$. So for f and g to be equal, the number b must be $b = 0$.

Exercise 3. Imagine the square in the coordinate plane with vertices $(-1,-1)$, $(-1,1)$, $(1,1)$ and $(1,-1)$. Can that be the graph of a function? Explain your answer.

Solution: Absolutely not, by the vertical line test.
In fact, if you take just any of the two vertical sides of the square, they are enough to show it cannot be the graph of a function. Indeed, any two different points on one of those sides have different y-coordinates but share the same x-value.

Exercise 4. Suppose that f is even and g is odd. Let h be the function defined by the expression $h(x) = 4f(g(2x))$. Is h even or odd? Explain your answer.

Solution:

$$h(-x) = 4f(g(-2x))$$

Since g is odd, $g(-2x) = -g(2x)$. So:

$$h(-x) = 4f(-g(2x))$$

But since f is even, $f(-g(2x)) = f(g(2x))$. Therefore:

$$h(-x) = 4f(g(2x)) = h(x)$$

Hence, h is even.

Exercise 5. Let $f(x) = \frac{2x+1}{3x-4}$. Find $f^{-1}(x)$, the expression for the inverse of f. Graph f and its inverse f^{-1}. For which x-values is f increasing? (You don't need to explain your answer for the last question.)

Solution: To find $f^{-1}(x)$, we need to solve the equation $f(x) = y$ and then the expression for x in terms of y will be $f^{-1}(y)$. If $f(x) = y$, then: $\dfrac{2x+1}{3x-4} = y$, and so:

$$2x + 1 = (3x - 4)y = 3x \cdot y - 4y$$

Regrouping the factors of x, we obtain:

$$2x - 3x \cdot y = -1 - 4y,$$

so that: $(2 - 3y)x = -1 - 4y$.
Therefore:

$$x = \frac{-1 - 4y}{2 - 3y} = \frac{-(1 + 4y)}{-(3y - 2)} = \frac{1 + 4y}{3y - 2}$$

In conclusion:

$$f^{-1}(x) = \frac{1 + 4x}{3x - 2}$$

To help yourself graph f, you can simplify its expression as follows:

$$f(x) = \frac{\frac{2}{3}(3x + \frac{3}{2}}{3x - 4} = \frac{2}{3}\left(\frac{3x - 4 + \frac{3}{2} + 4}{3x - 4}\right) = \frac{2}{3}\left(\frac{3x - 4}{3x - 4} + \frac{\frac{11}{2}}{3x - 4}\right) = \frac{2}{3}\left(1 + \frac{11}{2}\frac{1}{3x - 4}\right)$$

So essentially, f is a combination of stretches and shifts of the function defined by $1/x$. However, stretches and shifts do not change/affect the increasing or decreasing nature of a function. Only reflections can possibly do that. Since the function defined by $1/x$ is always decreasing, so is f and therefore f is **never** increasing.