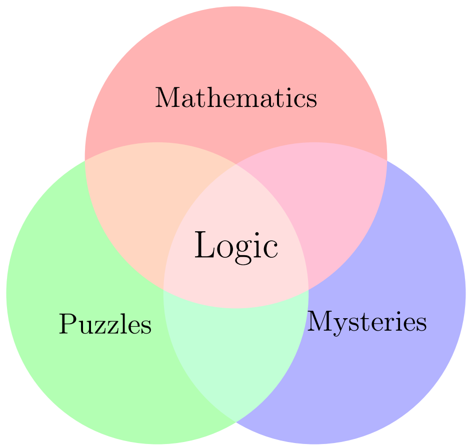
An Introduction to Logic

with Puzzles and Mysteries



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1 The Mathematics of Arguments

1.1 Arguments, Validity and Soundness

1. An *argument* is a series of statements (or *propositions*) in which one, called the *conclusion*, is meant to be a consequence of the others, called the *premises*.

2. An argument is *valid* if the conclusion must be true in any circumstance in which the premises are true. We say that the conclusion of a logically valid argument is a *logical consequence* of its premises.

3. An argument is *sound* if it and the premises are all true.

   Question 1. Can you think of an argument which is sound but not valid?

   Exercice 1. Classify the following arguments. Are they valid? Are they sound?

   (a) Anyone who wins an academy award is famous. Meryl Streep won an academy award. Hence, Meryl Streep is famous.

   (b) Harrison Ford is not famous. After all, actors who win academy awards are famous, and he never won one.

   (c) Mark Twain was born in Hannibal, Missouri, since Sam Clemens was born there, and Mark Twain *is* Sam Clemens.

1.2 Methods of Proof

(a) A proof of a statement S following from premises $P_1, P_2, \ldots, P_n$ is a detailed step-by-step demonstration which shows that S *must* be true in any circumstances in which the premises $P_1, P_2, \ldots, P_n$ are all true.

(b) Informal proofs and formal proofs only differ in style, not rigor. They both require precision and absolute certainty.

(c) To demonstrate the invalidity of an argument with premises $P_1, \ldots, P_n$ and conclusion S, we need to find a counterexample: a possible circumstance in which $P_1, P_2 \ldots, P_n$ are all true but S is false. Such a counterexample shows that S is not a consequence of the premises $P_1, P_2, \ldots, P_n$.

Exercice 2. In the previous exercise (**Exercise 1**), find out which premises were not true in whichever arguments were not valid.

Exercice 3. Find the truth values of the following statements.

- $\neg P \wedge Q$.
- $P \vee \neg Q$.

1.3 The Mystery of Today

One evening, Sherlock Holmes and Dr. Watson were sitting by the fireplace discussing the increasing crime rate in England. Suddenly their discussion was interrupted by a gentle knock at the door.

"There's a Mrs. Crowfield to see you, Mr. Holmes. She says that she needs your help desperately," said Mrs. Hudson.
"Will you show her in, please," answered Holmes, throwing a significant glance at Mr. Watson.

The visitor turned out to be a young woman. She seemed agitated and started talking immediately as she rushed into the room. The woman asked the two men to come with her to her manor, which was a short ride north from London. Holmes and his friend could not refuse to help a lady so they quickly took everything they might need and went to the railway station.

On their way to the manor Mrs. Crowfield told the men about the crime that had been committed in their house. Someone had broken into her husband's study and destroyed the invention he had been working on for the last several years. Mr. Crowfield turned out to be a scientist and when he had seen his work destroyed, he had had a heart attack, but luckily he was still alive and there was hope for recovery. Mrs. Crowfield couldn't figure out what anybody had to gain by that hostile act. Was it just vandalism, done out of sheer spite?

Anyway, when Holmes arrived at the manor he discovered that there were 5 suspects:

1. Jacob, Mr. Crowfield's brother,
2. The butler,
3. Mrs. Crowfield's maid,
4. The gardener,
5. The cook.

Holmes questioned each subject. They each made three statements, **two of which were true and one of which was false.** And the guilty one was revealed. How quickly can you find the truth?

1. The butler said:
   (a) I am innocent.
   (b) I have never destroyed anything on purpose.
   (c) The maid did it.
2. Jacob said:
   (a) I did not do the damage.
   (b) The damage was done by someone who really hates my brother.
   (c) Nobody helped my brother in his work.
3. The gardner said:
   (a) I am innocent.
   (b) I never talk to the cook.
   (c) The maid is guilty.
4. The maid said
   (a) I did not destroy the invention.
   (b) The cook did it.
   (c) The butler did not tell the truth when he said I did it.
5. The cook said:
   (a) I am innocent.
   (b) Mr. Jacob is guilty.
   (c) The gardener and I are old friends.

Answer: Jacob is the culprit!

2 Rules of Inference

Remark 1. Parentheses must be whenever ambiguity would result from their omission. This means that conjunctions and disjunctions must be "wrapped" in parentheses whenever combined by means of some other connective. Think of how parentheses are used with addition and multiplication. For example, the conjunction $P \wedge Q \wedge R$ can be written as $(P \wedge Q) \wedge R$ or as $P \wedge (Q \wedge R)$ and this will matter when we do truth tables. We will want to think as whichever conjunction between brackets as a separate proposition.

2.1 Conditionals

1. The setence $P \to Q$ is only false when P is true and Q is false. Otherwise, it is true.
2. To say that Q is a logical consequence of $P_1, \ldots, P_n$ is the same as saying that the sentence $P_1 \wedge \cdots \wedge P_n \to Q$ is always true.
3. The sentence $P \leftrightarrow Q$ is true exactly when P and Q have the same truth values.

2.2 Rules of Inference

1. Modus Ponens:
   - Premises: $P, P \to Q$.
   - Conclusion: Q.
2. Modus Tollens:
   - Premises: $\neg Q, P \to Q$.
   - Conclusion: $\neg P$.
3. Hypothetical Syllogism:
   - Premises: $P \to Q, Q \to R$.
   - Conclusion: $P \to R$.
4. Disjunctive Syllogism:
   - Premises: $\neg P, P \vee Q$.
   - Conclusion: Q.
5. Addition:
   - Premise: P.
   - Conclusion: $P \vee Q$.
6. Simplification:
   - Premises: $P \wedge Q$.
   - Conclusion: P.
7. Conjunction:
   - Premises: P, Q.
   - Conclusion: $P \wedge Q$.
8. Resolution:
   - Premises: $P \vee Q, \neg P \to R$.
   - Conclusion: $Q \vee R$.

Exercice 4. We know that:

1. It is not sunny this afternoon, and it is colder than yesterday.
2. We will go swimming only if it is sunny.
3. If we do not go swimming, we will play basketball.
4. If we play basketball, we will go home early.

Can you conclude that: "We will go home early" from these premises? Symbolize the premises as sentences composed of propositions and use the Fitch format.

Exercice 5. It is known that:

1. If you send me an email, then I will finish my program.
2. If you do not send me an email, then I will go to sleep early.
3. If I go to sleep early, I will wake up refreshed.

Can you conclude "If I do not finish my program, then I will wake up refreshed" from these premises? Again, symbolize appropriately and use the Fitch format.

Exercice 6. Consider the following three hypotheses:

1. Alan likes kangaroos, and either Betty likes frogs or Carl likes hamsters.
2. If Betty likes frogs, then Alan doesn't like kangaroos.
3. If Carl likes hamsters, then Betty likes frogs.

Write a clear argument to show that these three hypotheses are contradictory.

2.3 The Puzzle of Today

Here is a logical puzzle invented by Lewis Carroll. The premises are:

1. No interesting poems are unpopular among people of real taste.
2. No modern poetry is free from affectation.
3. All your poems are on the subject of soap-bubbles.
4. No affected poetry is popular among people of real taste.
5. No ancient poem is on the subject of soap-bubbles.

What can you conclude about *your* poetry, according to Lewis Carroll?